How to make loads of money using persistent homology

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You can find these slides at `jses.site/talks`. This is not my own work.

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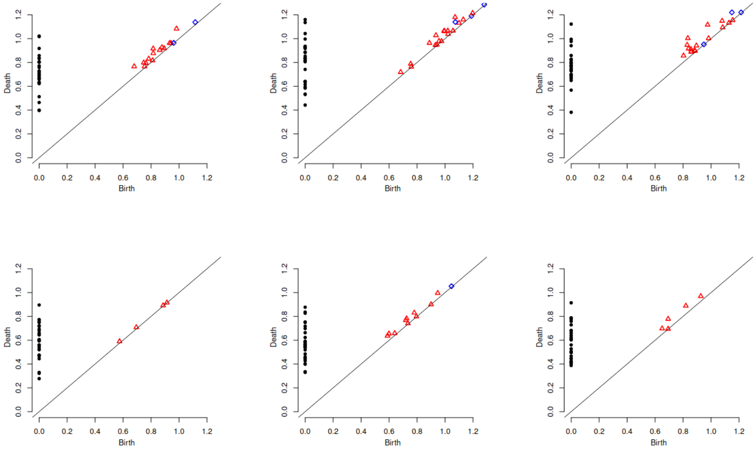
Nasty Formula (for Pearson correlation coefficient)

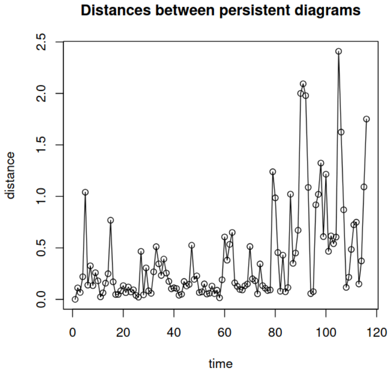
For fixed T (author uses $T = 15$),

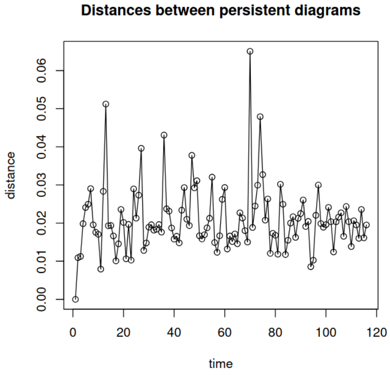
$$c_{i,j}(t) := \frac{\sum_{\tau=t-T}^{t} (x_i(\tau) - \overline{x_i})(x_j(\tau) - \overline{x_j})}{\sqrt{\sum_{\tau=t-T}^{t} (x_i(\tau) - \overline{x_i})^2} \sqrt{\sum_{\tau=t-T}^{t} (x_j(\tau) - \overline{x_j})^2}}.$$

Some Graphs

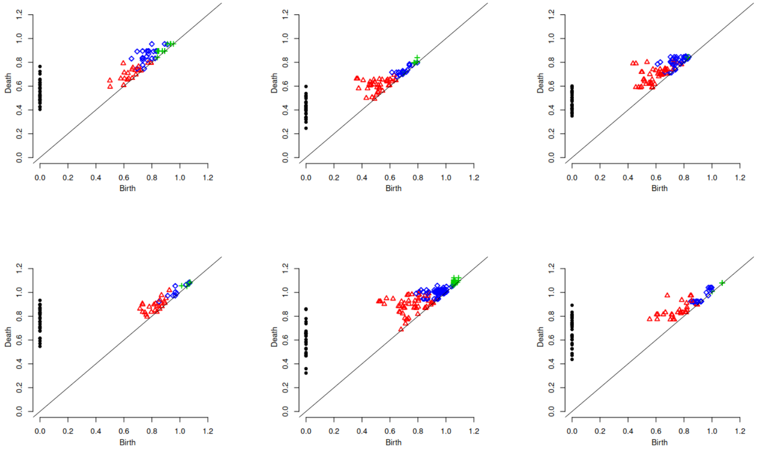


Some More Graphs

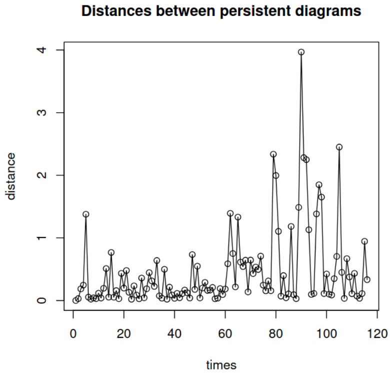


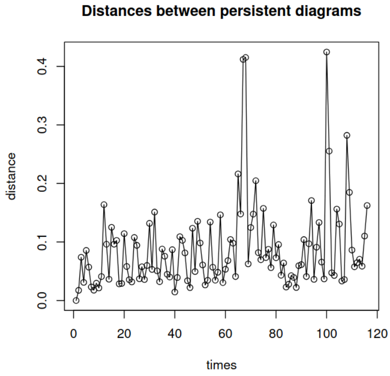


Further Graphs



Yet More Graphs





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- Futures – buy and agree to sell a stock at a fixed time in the future.
- Authors define an “abnormality index” based on the price of futures.

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$$\begin{aligned}
\delta_i &:= i\text{th Norm} - \text{EMA}_{i-1}, \\
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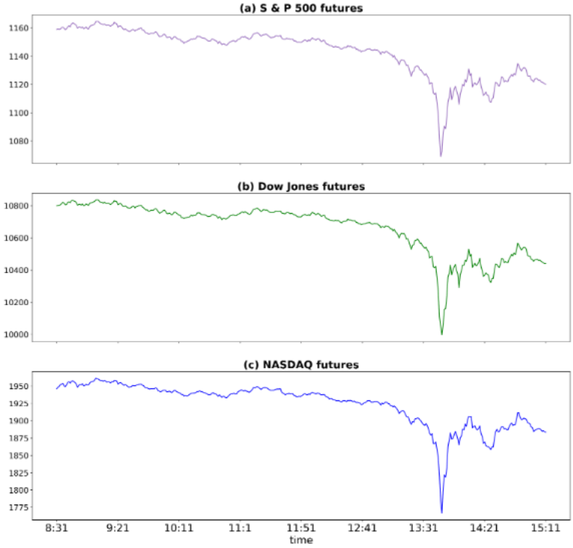
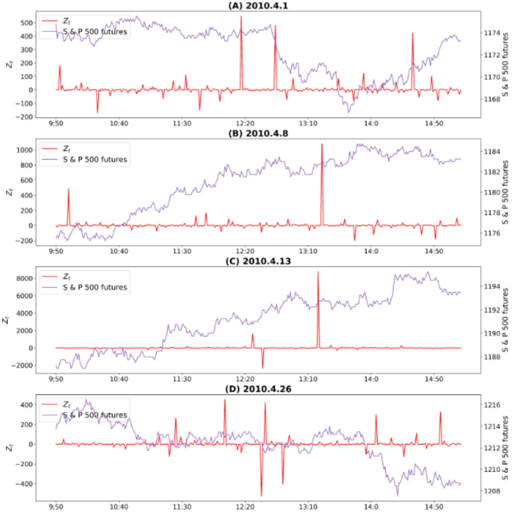
- $Z_t = \frac{Y_t - \mathrm{EMA}_{t-1}}{\mathrm{EMVar}_{t-1}}$.

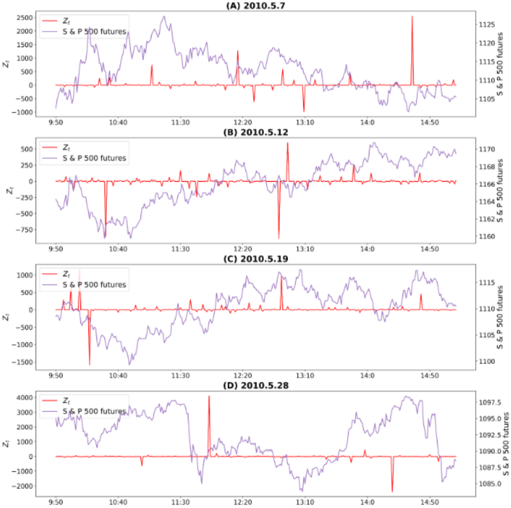
Surely not more graphs?!?



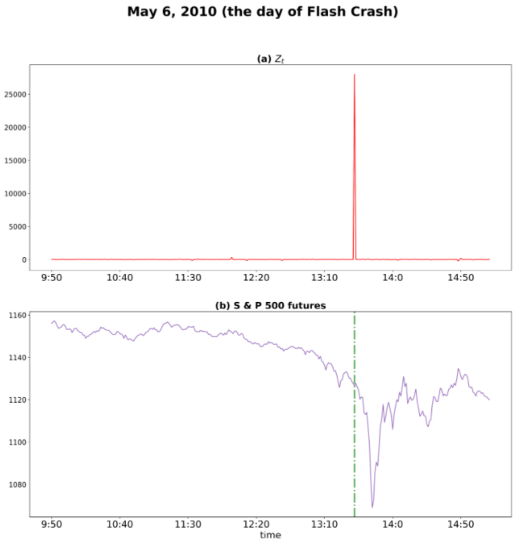
Pre-Event Graphs



Post-Event Graphs



I can't believe it's graphs



References I

- Marian Gidea, *Topology data analysis of critical transitions in financial networks*, 2017.
- Wonse Kim, Younng-Jin Kim, Gihyun Lee, and Woong Kook, *Investigation of flash crash via topological data analysis*.
- Miguel A. Ruiz-Ortiz, José Carlos Gómez-Larrañaga, and Jesús Rodríguez-Viorato, *A persistent-homology-based turbulence index and some applications of tda on financial markets*, 2023.